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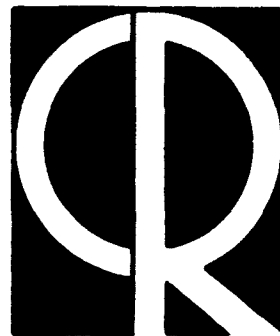
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**Research Report**

## **Cancellation of Doppler Frequency Shift**

**RICHARD M. WAETJEN**

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**Research Report**

## **Cancellation of Doppler Frequency Shift**

**RICHARD M. WAETJEN**

## Abstract

A circuit is described which is capable of cancelling first-order Doppler frequency shifts. It has potentialities in the fields of communication, data transmission, and navigation systems for vehicles moving at high velocities. The circuit has the following features:

I. Received and transmitted frequencies at each of the stations involved are different, thus preventing feedback from the transmitting to the receiving antenna at the stations.

II. Amplification of the signals is provided at a frequency which can be chosen to have a convenient value. This factor is important in the case of microwave systems, since high-gain microwave amplifiers are much more expensive than, and not as reliable as, IF amplifiers operating at frequencies in the 1 to 300 megacycle range.

III. The system is capable of continuous-wave operation and simultaneous transmission and reception at each of the communicating stations.

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## Cancellation of Doppler Frequency Shift

### 1. INTRODUCTION

The described circuit is capable of cancelling first-order Doppler frequency shift which is proportional to  $\frac{v}{c}$ , where  $v$  is the velocity of the two communicating stations relative to each other, and  $c$  is the velocity of electromagnetic wave propagation.

Second-order Doppler which is of the form  $\frac{v^2}{c^2}$  cannot be cancelled by this circuit. The magnitude of the  $\frac{v^2}{c^2}$  term is of course quite small at low velocities. Table 1 gives some values for  $\frac{v}{c}$  and  $\frac{v^2}{c^2}$  for various velocities.

Table 1

$v$ m/sec	type vehicle	$\frac{v}{c}$ first-order Doppler	$\frac{v^2}{c^2}$ second-order Doppler
100	propeller aircraft	$3 \times 10^{-7}$	$10^{-13}$
300	subsonic jet aircraft	$10^{-6}$	$10^{-12}$
1000	Mach 3 jet aircraft	$3 \times 10^{-6}$	$10^{-11}$
6000	Earth satellite	$2 \times 10^{-5}$	$4 \times 10^{-10}$

(Author's manuscript approved for publication 31 January 1963)

It is apparent from Table 1 to what extent the influence of Doppler can be eliminated by the use of the circuit described below. Practical applications, however, may not be so obvious. Typical uses for the circuit will be in the field of fast moving vehicles like satellites and spacecraft, although the original application for which the scheme had been designed was the transmission of data from a balloon to the ground. This and possible future applications are described in the appendix.

It is also obvious from Table 1 that even second-order Doppler can still assume a sizable amount, especially at velocities typical with supersonic aircraft and with space vehicles.

The actual circuit is described in detail in the following section.

## 2. DESCRIPTION OF CIRCUIT

The frequency of a signal transmitted from one station to another experiences a Doppler shift if one station is moving relative to the other. If two-way transmission is employed, this Doppler shift can be cancelled except for a second-order term according to Zacharias (Refer to Figure 1).<sup>1</sup>

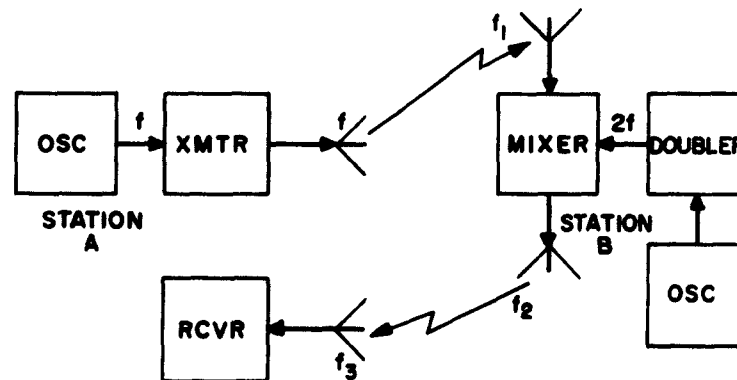


Figure 1

Both stations generate a frequency  $f$ . Station A transmits  $f$ . Station B is moving with respect to A at a velocity  $v$ . The velocity,  $v$ , is positive or negative

if B moves toward or away from A, respectively, and it is the velocity component along the direct path A - B. The frequency  $f_1$  received at station B is equal to:

$$f_1 = f \left( 1 \pm \frac{v}{c} \right) . \quad (1)$$

At station B,  $f$  is doubled and  $f_1$  is subtracted from  $2f$ , the difference frequency being  $f_2$ .

$$\begin{aligned} f_2 &= 2f - f \left( 1 \pm \frac{v}{c} \right) \\ &= f \left( 1 \mp \frac{v}{c} \right) \end{aligned} \quad (2)$$

The frequency  $f_2$  is transmitted from B. On its way to A, this signal undergoes a Doppler shift, and A receives  $f_3$ .

$$\begin{aligned} f_3 &= f_2 \left( 1 \pm \frac{v}{c} \right) = f \left( 1 \mp \frac{v}{c} \right) \left( 1 \pm \frac{v}{c} \right) \\ &= f \left( 1 - \frac{v^2}{c^2} \right) \end{aligned} \quad (3)$$

Thus,  $f_3$  is equal to the originally transmitted frequency  $f$ , and the Doppler frequency shift was effectively cancelled except for a second-order term  $f \frac{v^2}{c^2}$ . It can also be noted that this term is always negative, making  $f_3$  smaller than  $f$ , independent of whether B is moving towards or away from A.

This second-order Doppler term is very small, and it can be neglected in many cases. There are, however, cases where it can be of significance. Two such instances are described in the appendix.

The block diagram in Figure 1 is greatly simplified. Amplifiers and filters have been omitted.

In practice, this system is not going to work unless it is modified, because at station B the frequency  $f_2$  is almost equal to  $f_1$  (plus or minus  $2 f \frac{v}{c}$ ), and  $f_2$  will be fed back to the mixer input. In addition to this,  $f_1$  will also be present at the output of the mixer, because the output of the mixer consists of both the input frequencies and their sum and difference frequencies. Frequency  $f_2$  could be separated from  $f_1$  by means of filtering only if the Doppler frequency shift were very large.

To overcome these problems, the following circuit modifications are proposed below.<sup>2,3</sup>

The mixer at station B is replaced by two mixers and an amplifier, as shown in Figure 2. The mixer injection frequencies  $f'$  and  $f''$  are synthesized from  $f$ , satisfying the following conditions:

$$f' + f'' = 2f; af = f'; (2 - a)f = f'' . \quad (4)$$

The output of the first mixer is equal to  $f' - f_1$ . When  $f''$  is added in the second mixer, the output is then:

$$f_2 = f' + f'' = 2f - f_1 ,$$

which is equal to the frequency one would have produced with a single mixer by subtracting  $f_1$  from  $2f$ , but now all spurious signals including  $f_1$  have been removed.

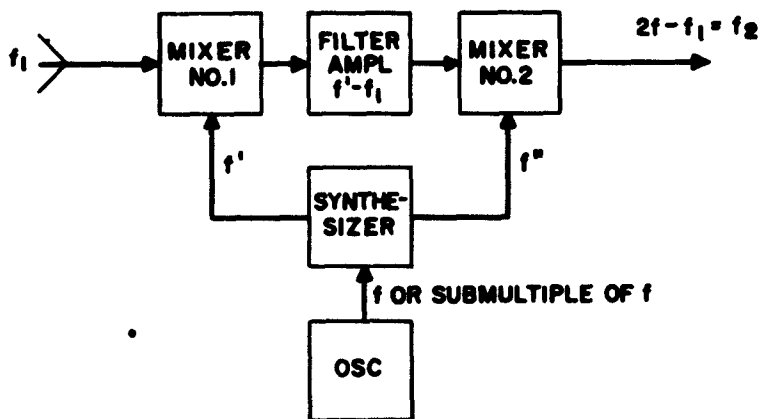


Figure 2

Frequency  $f_2$ , of course, is still very close to  $f_1$ , and if retransmitted, could be fed back to the first mixer and thus upset the proper operation of the system.

The frequency which is transmitted from station B has to be sufficiently offset from  $f_1$  to fall outside the passband of the filter amplifier. The only way to accomplish this, and at the same time not to upset the Doppler cancelling feature of the circuit, is to multiply  $f_2$  by a factor  $n$  which is not equal to one. Frequency  $f_2$ , Eq.(2), is equal to  $f(1 \mp \frac{v}{c})$ . If  $f_2$  is multiplied by  $n$ , one obtains:

$$n f_2 = n f(1 \mp \frac{v}{c}) . \quad (5)$$

This is again multiplied by  $(1 \pm \frac{v}{c})$  as the signal propagates to A and yields:

$$f_3 = nf(1 \mp \frac{v}{c})(1 \pm \frac{v}{c}) = nf(1 - \frac{v^2}{c^2}) . \quad (6)$$

Thus, Doppler frequency shift has been cancelled, although now the transmitted and received frequencies at stations A and B are no longer equal.

In a practical system, one would choose the value for  $n$  close to one (11/10, 9/10, 9/8, etc.), to eliminate, or reduce to a minimum, frequency shifts which can be caused by frequency dependent propagation anomalies.

It is not possible to offset  $f_2$  by frequency addition or subtraction and still maintain the Doppler cancelling feature of the system, unless the added frequency is derived from  $f_2$  alone; for instance, by dividing  $f_2$  by  $m$ . This latter method would then yield:

$$f_2 + \frac{f_2}{m} = f_2(1 + \frac{1}{m}) . \quad (7)$$

If we call  $n = 1 + \frac{1}{m}$ , we immediately recognize that this procedure is equivalent to the one described in the previous paragraphs, Eq.(5) and Eq.(6), and Doppler is still being cancelled. If, however, the added frequency is derived from  $f$  and is equal to  $pf$ , one obtains:

$$f_2 + pf = f(1 \mp \frac{v}{c}) + pf = f(1 \mp \frac{v}{c} + p) . \quad (8)$$

This signal undergoes a Doppler shift when propagating to A and yields.

$$f_3 = f(1 \mp \frac{v}{c} + p)(1 \pm \frac{v}{c})$$

$$f_3 = f(1 - \frac{v^2}{c^2}) + pf(1 \pm \frac{v}{c}). \quad (9)$$

It is apparent from this that Doppler frequency shift has not effectively been cancelled.

The term  $pf(1 \pm \frac{v}{c})$  in Eq.(9) can have a relatively small value if  $p$  is a small number, and for some applications it might be sufficient to produce the necessary frequency offset by this method, because the  $f\frac{v}{c}$  term is still being eliminated. The following typical numbers may serve to illustrate this possibility.

Consider a signal frequency,  $f$ , of 1000 Mc, a predetection bandwidth of 1 Mc, and a satellite with a velocity,  $v$ , of 8000 m/sec. The first-order Doppler then is equal to  $f\frac{v}{c} = 10^9 \times 2 \times 10^{-5} = 2 \times 10^4$  cps, and second-order Doppler is equal to  $f\frac{v^2}{c^2} = 10^9 \times 4 \times 10^{-10} = 0.4$  cps. In this case,  $p$  will have to be large enough to offset  $f_2$  sufficiently from  $f_1$  to prevent overloading of the mixer and IF amplifier (filter amplifier in Figure 2). Assuming a five megacycle offset,  $pf\frac{v}{c} = 5 \times 10^6 \text{ cps} \times 2 \times 10^{-5} = 100$  cps, a value which is still two orders of magnitude below the magnitude of the first-order Doppler, and which might be sufficiently low for some systems' applications. This method can certainly not explore the full capabilities of the Doppler cancellation scheme, as the  $pf\frac{v}{c}$  term is almost three orders of magnitude higher than the second-order Doppler term in this particular example.

The block diagram (Figure 3) combines and illustrates the previously outlined ideas. The "times  $n$ " block will consist of a divider-multiplier-type network if  $n$  is chosen close to one and not an integer. Such circuits, employing regenerative, varicap, locked oscillator, or phantastron frequency dividers, work quite well at frequencies of several megacycles. Some of these dividers will operate up to 50 Mc or even 100 Mc and beyond, but they have not been reported to operate at much higher UHF or microwave frequencies, although such operation seems theoretically quite feasible. It appears, therefore, that the circuit shown in Figure 3 is of hypothetical rather than practical value because it does not work in the kilomegacycle range where Doppler shifts may become quite large.

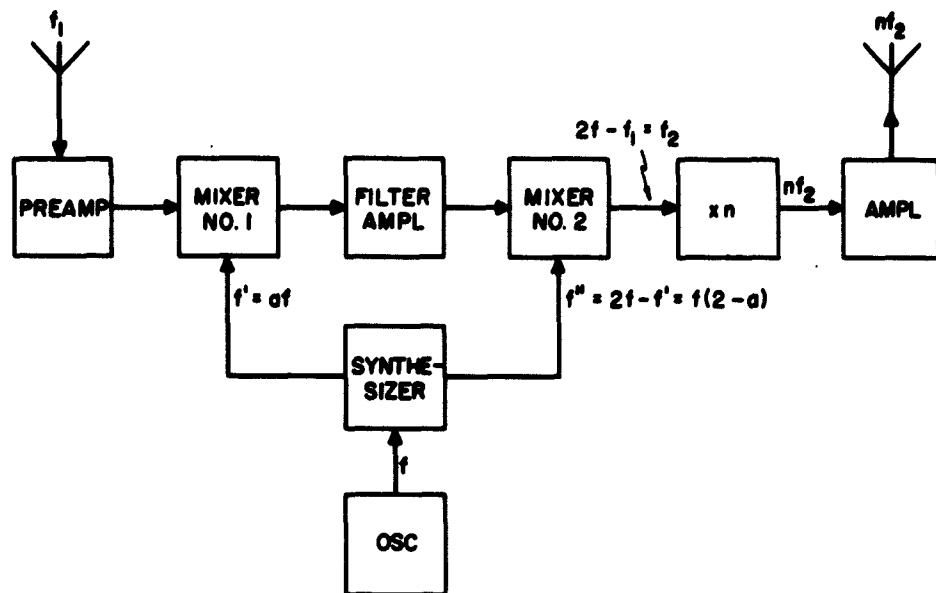


Figure 3

From the aforesaid, it is apparent that this system is capable of cancelling a Doppler frequency shift if the frequency offset necessary for continuous-wave two-way transmission is obtained by multiplying a frequency containing the  $(1 \pm \frac{v}{c})$  term by a factor,  $n$ . This means that the multiplication does not necessarily have to be performed at a place following the second mixer; that is, at possibly a very high frequency. In Figure 4, the "times  $n$ " network is placed before the second mixer, where the frequency can be chosen to have a conveniently low value, and frequency division is no problem. The operation of this circuit is explained as follows:

The frequency received from station A at station B is equal to  $f(1 \pm \frac{v}{c})$ . This is subtracted from  $f' = af$ , where the factor  $a$  is greater than one, producing  $f(a - 1 \pm \frac{v}{c})$ . After passing through the filter amplifier, this signal is multiplied by  $n$ . The resultant signal  $nf(a - 1 \pm \frac{v}{c})$  is fed to the second mixer. In the second mixer we have to add a frequency to produce the desired output of

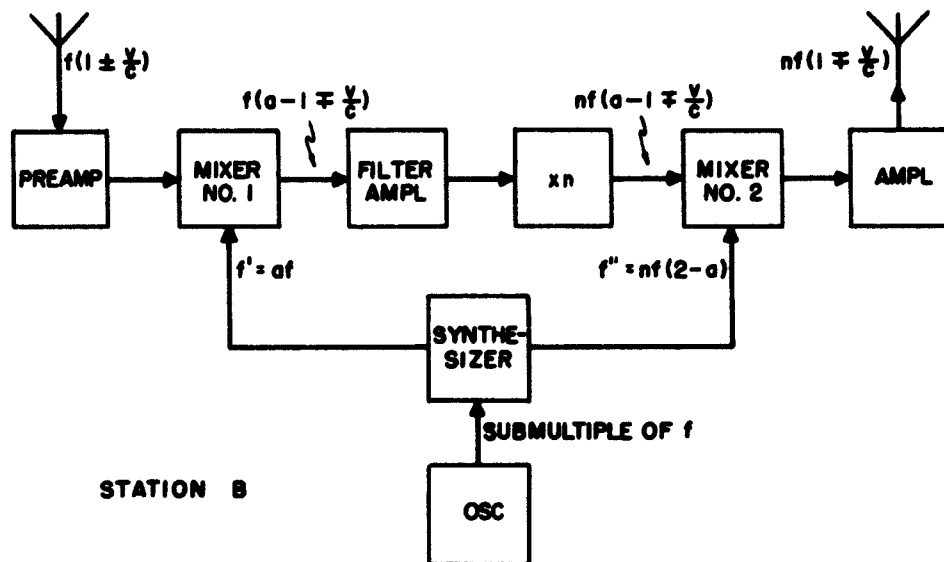


Figure 4

$nf_2 = nf(1 \pm \frac{V}{C})$ . Thus, the frequency to be added must be:

$$nf(1 \pm \frac{V}{C}) - nf(a - 1 \pm \frac{V}{C}) = nf(2 - a). \quad (10)$$

This happens to be equal to  $n$  times  $f''$ , according to Eq.(4). If the values for factors  $a$  and  $n$  are chosen properly, the mixer injection frequencies can be generated simply by frequency multiplication from a common source; for instance, a crystal oscillator operating at a submultiple frequency of  $f$ . A practical example is chosen to illustrate this feature:

$$f = 240 \text{ Mc}, \quad n = 7/8, \quad nf = 210 \text{ Mc}, \quad a = 14/13.$$

The frequency at the output of the first mixer is then equal to  $(14/13 - 1)f = f/13 = 18.46 \text{ Mc}$ , which can be divided by eight and multiplied by seven. The

frequency at the output of the "times n" network is equal to  $7f/13 \times 8 = 16.15$  Mc, which is brought to the second mixer. Here we add  $nf(2 - a) = 21f/26$  and obtain  $7/8$  times  $f$ . In this case, the frequency synthesizer would be quite simple (See Figure 5). The output of the crystal oscillator operating at  $(1/26)f = 9.23$  Mc is multiplied by seven and yields  $7f/26 = 64.615$  Mc. Multiplying this by four, we obtain the first mixer injection frequency,  $14f/13 = 258.46$  Mc. The injection frequency for the second mixer is obtained through multiplication by three,  $21f/26 = 193.85$  Mc.

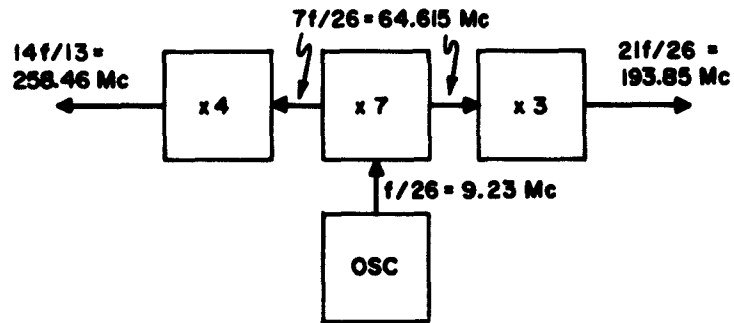


Figure 5

Figure 6 is a block diagram showing the complete circuit at station B for the chosen example.

In a number of cases, it will not be necessary to have transmitted and received frequencies close together in order to avoid frequency shifts due to propagation anomalies. Such shifts would be very small at microwave frequencies, if present at all, and would probably be negligible in many instances. If  $f_2$  is a multiple of  $f_1$ , the circuit simplifies considerably because frequency division is not necessary;  $n$  would be an integer, and the "times n" network would simply consist of a multiplier. The circuit is simplified even more if  $f'$  and  $f''$  are equal. Where  $n = 2$ , for instance, this is the case if  $a = 4/3$ , and where  $n = 3$ , the factor  $a$  has to be equal to  $3/2$ .

These special cases, however, do not explore one of the advantages of the circuit, which is signal amplification at a relatively low frequency. With  $a = 4/3$ , the output of the first mixer is equal to  $f/3$ . This value might be much too high

for convenient amplification as in the case of microwave systems. One would have to resort to traveling-wave-tube amplifiers, which are expensive.

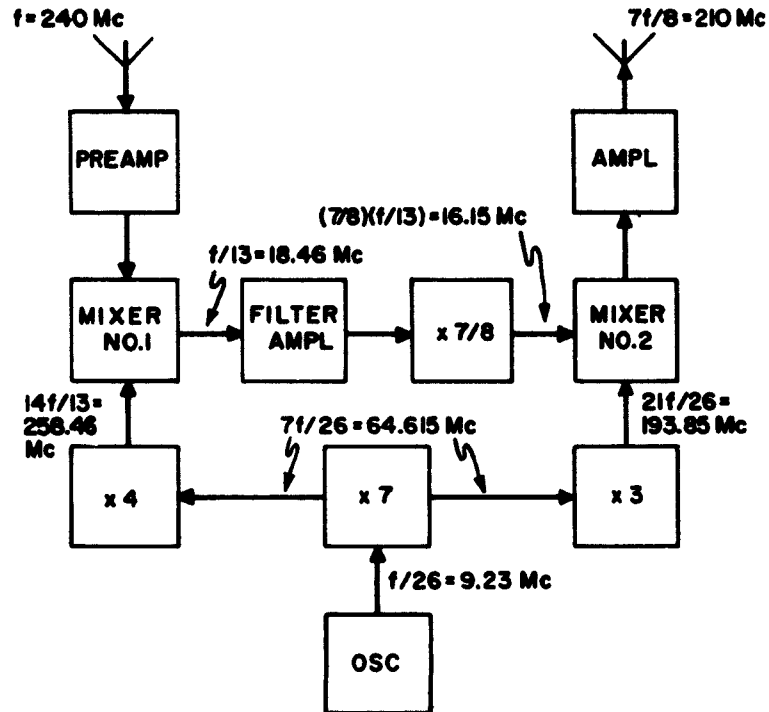


Figure 6

Other simple-to-realize combinations of  $f'$  and  $f''$  are those in which they are both multiples of a common frequency, as illustrated in the above UHF example in Figure 6. Once the frequencies for transmission and reception have been determined (thus establishing a value for  $n$ ), the factor  $a$  can be chosen simply from the point of view of obtaining the least complex system regarding the convenient generation of  $f'$ ,  $f''$ , and a suitable frequency at the first mixer

output, at which amplification can be performed.

Up to this point it has been assumed that the frequencies generated by the oscillators in stations A and B are identical and equal to  $f$ . This assumption is of course unrealistic. There is no absolutely stable oscillator. Even atomic or molecular oscillators or clocks exhibit a measurable drift, although in many practical cases such drift will be insignificant.

The effect of frequency drift can be evaluated by means of simple algebra, referring to Figure 1, and Eq.(1), Eq.(2), and Eq.(3). This derivation will hold exactly with the more complex circuits depicted in Figures 3 through 6. The case where the frequency of the oscillator at station A has drifted is considered first. The frequency  $f(1 + \Delta)$  is transmitted instead of the correct frequency  $f$ ,  $\Delta$  being a number much smaller than one. Frequency  $f_1$  is then equal to:

$$f(1 + \Delta)(1 \pm \frac{v}{c}) = f(1 \pm \frac{v}{c} + \Delta \pm \Delta \frac{v}{c}) = f_1 \quad (1a)$$

Doppler

This is subtracted from  $2f$  at station B to yield:

$$\begin{aligned} f_2 &= 2f - f(1 \pm \frac{v}{c} + \Delta \pm \Delta \frac{v}{c}) \\ &= f(1 \mp \frac{v}{c} - \Delta \mp \Delta \frac{v}{c}) \end{aligned} \quad (2a)$$

The frequency received at A becomes:

$$\begin{aligned} f_3 &= f_2 (1 \pm \frac{v}{c}) \\ f_3 &= f(1 - \frac{v^2}{c^2} - \Delta \mp \Delta \frac{v^2}{c^2} - \Delta \mp 2\Delta \frac{v}{c}) \end{aligned} \quad (3a)$$

In many practical cases it will be possible to neglect the  $\Delta \frac{v^2}{c^2}$  and  $2\Delta \frac{v}{c}$  terms because they are much smaller than  $\frac{v^2}{c^2}$  and  $\Delta$  respectively, provided that  $\frac{v}{c}$  and  $\Delta$  are much smaller than one. Eq.(3a) thus reduces to:

$$f_3 = f(1 - \frac{v^2}{c^2} - \Delta) \quad (3a)$$

Therefore the frequency received at station A is smaller than the frequency  $f$  by an amount equal to  $\Delta f$ , and smaller than the transmitted frequency  $f(1 + \Delta)$  by an amount equal to  $2\Delta f$ .

Now let us consider a frequency drift at station B. The oscillator at B is generating a frequency equal to  $f(1 + \Delta)$ . Station A transmits  $f$ . Station B receives:

$$f_1 = f(1 \pm \frac{v}{c}), \quad (1b)$$

and transmits:

$$\begin{aligned} f_2 &= 2f(1 + \Delta) - f(1 \pm \frac{v}{c}) \\ &= f(1 \mp \frac{v}{c} + 2\Delta). \end{aligned} \quad (2b)$$

This arrives at A as:

$$\begin{aligned} f_3 &= f(1 \mp \frac{v}{c} + 2\Delta)(1 \pm \frac{v}{c}) \\ &= f(1 - \frac{v^2}{c^2} + 2\Delta \pm 2\Delta \frac{v}{c}) \end{aligned} \quad (3b)$$

As in Eq.(3a) above, we shall neglect the  $2\Delta \frac{v}{c}$  term, and Eq.(3b) reduces to:

$$f_3 = f(1 - \frac{v^2}{c^2} + 2\Delta). \quad (3b)$$

When comparing Eq.(3a) and Eq.(3b), one detects that the frequency received at station A is offset from the transmitted frequency by an amount equal to  $2\Delta f$ , regardless of whether or not the oscillator frequency at A or at B has drifted. Frequency  $f_3$  is higher than  $f$  as transmitted from A, if the oscillator frequency at B has increased. Frequency  $f_3$  is lower than  $f$  as transmitted from A, if the frequency of the oscillator at A has increased.

### 3. CONCLUSION

First-order Doppler frequency shift, as occurring in the case of communicating stations which are in motion with respect to each other, can be cancelled with the circuit as described in the preceding paragraphs.

Second-order Doppler shift cannot be cancelled with this circuit.

It must be noted that the Doppler cancelling feature of this circuit is effective only if the signal makes a round trip from station A to station B and back to station A. The frequency of the signal received at station A does not vary, regardless of the relative velocity of the two stations, except for second-order Doppler.

The frequencies generated by oscillators in stations A and B (called  $f$  on the preceding pages) must be equal, or as close to being equal as possible, for the system to operate properly. On the other hand, if one very stable oscillator is at station B, and an inherently unstable one is at A, then the one at A can be adjusted and, for instance, positively controlled by means of an automatic frequency control circuit, until the frequencies transmitted and received at A are equal (and presumably equal to the stable oscillator frequency at B). Thus the above described circuit can also be employed to transmit a standard frequency (or time) from one point to another with great accuracy, while the stations are in motion with respect to each other.

Applications of this circuit are mainly in the fields of space navigation, guidance, and tracking. They could also be used in data transmission, and possibly communications, in cases where the Doppler frequency shift limits the system performance.

## Appendix

### 1. THE AFCRL GRAVITATIONAL REDSHIFT EXPERIMENT

In 1959 and 1960, the Communication Sciences Laboratory initiated an experimental effort in order to prove Einstein's Principle of Equivalence.

It is beyond the scope of this paper to explain or describe in detail the Principle of Equivalence. The effect which the AFCRL experiment was to detect was the so-called Clock Paradox, a phenomenon predicted by Einstein when he combined the Principle of Equivalence with Doppler's Principle. This phenomenon is that two identical clocks, one of which is under constant acceleration relative to the other, run at different rates. Since, according to the Principle of Equivalence, the force of a uniform gravitational field and the force resulting from constant acceleration are indistinguishable, identical clocks which are placed in different gravitational fields also run at different rates.

It was planned to operate two atomic clocks, one of them on the ground, the other at an elevation of 100,000 feet inside a balloon gondola, and to measure the effect of the gravitational field by comparing the output frequencies of the two clocks. The clock at the lower gravitational potential at 100,000 feet elevation would run at a slower rate. Theoretically it would lag one microsecond in three days relative to the clock on the ground. The magnitude of the effect (about three parts in  $10^{12}$ ) required the use of clocks with a stability of a few parts in  $10^{13}$  in three days. The use of the Atomichron, manufactured by the National Company, Inc., of Malden, Massachusetts, was anticipated because it

was the most stable clock available commercially at the time. It turned out, however, that even for this clock, the required stability could not be guaranteed under the specified environmental conditions. This was the reason why the AFCRL experiment was postponed until clocks with the necessary stability became available. The experiment was cancelled altogether when Professor Pound measured the Clock Paradox at Harvard University, thus obviating further experiments.<sup>5</sup>

There was a third reason for hesitation in the actual instrumentation and performance of the experiment. This was the fact that it could never be guaranteed within reasonable limits that the airborne package with a number of very expensive instruments could be recovered intact, or even recovered at all. Between three and ten balloon flights had been anticipated, which might have turned out to be very expensive because of the necessary equipment repairs and replacements.

Much work went into this project; as a result of this, techniques were developed which can be applied to other problems requiring similar high-precision time or frequency measurements.

The circuit described on the earlier pages of this report was developed for the Gravitational Redshift Experiment. Station A was to be ground-based,  $f$  being generated by an Atomichron oscillator. Station B was to operate in a balloon gondola at an altitude of 100,000 feet,  $f$  being generated by a second Atomichron. The balloon was to stay aloft for a period of three days, during which time the system was to operate continuously. Any deviation,  $\Delta f$ , of the airborne oscillator would record as  $2\Delta f$  at the ground receiver. The Doppler cancellation scheme had to be employed because the balloon could not be kept stationary. Winds at high altitudes can reach velocities in excess of 100 m/sec. Considering a 100 m/sec wind, first-order Doppler can be as high as:

$$\frac{v}{c} = 3.3 \times 10^{-7} \text{ or three parts in } 10^6,$$

a value which is six orders of magnitude higher than the gravitational effect of about three parts in  $10^{12}$ .

The second-order Doppler shift for  $v = 100$  m/sec is equal to:

$$\frac{v^2}{2c^2} = 1 \text{ part in } 10^{13}.$$

This means that even second-order Doppler cannot be neglected in this measurement. It could assume approximately the same magnitude as the anticipated

minimum frequency instability of the Atomichron. It could not be corrected or controlled, although it might have been possible to do the experiment on days on which low wind velocities at high altitudes prevailed.

From Eq.(2b) and Eq.(3b) on page 12, it is apparent that a drift of the airborne oscillator frequency,  $\Delta f$ , would reflect in a frequency offset of  $2\Delta f$  in  $f_3$ , the frequency received at station A. Thus, if the airborne oscillator frequency drifted due to the influence of the lower gravitational field, twice this amount could be detected and measured on the ground by comparing it with the frequency  $f$  as generated in the ground frequency standard. The frequencies used for ground-to-air and air-to-ground transmissions were approximately those indicated in Figure 6. The actual frequency of the air-to-ground link was 247 Mc. At this frequency, the Gravitational Redshift  $2\Delta f$  would have amounted only to 65 cps in three days. The fairly complex instrumentation used to measure this quantity is described in a separate paper.<sup>4</sup>

## 2. THE SECOND-ORDER DOPPLER SHIFT

As indicated above, even relatively low velocities, of the order of 100 m/sec or approximately 200 knots, can produce second-order Doppler, which under certain conditions is large enough to cause concern. Figure 7 illustrates what values this second-order term can assume at higher vehicle velocities. It is obvious that at a velocity of 50,000 m/sec, which may be encountered during interplanetary travel, the second-order Doppler term has a magnitude of  $3 \times 10^{-8}$ . This 50,000 m/sec velocity, incidentally, is quite realistic if one considers that the earth itself is traveling in its orbit around the sun at a mean velocity of almost 30,000 m/sec, and that relative vehicle velocities higher than this can occur under certain conditions. If one wants to transmit a frequency to the interplanetary vehicle with an accuracy greater than  $3 \times 10^{-8}$ , (for instance, for navigational purposes), one would have to cancel or reduce even the second-order Doppler term.

At this point, one should consider that most, if not all, radio navigation, tracking, and guidance systems depend to a high degree on the accurate measurement, or the accurate knowledge, of a frequency. If radio navigation is employed in a space mission, the accuracy and stability of the reference frequency will have to be much greater than those involved in most comparable global navigation systems because of the greater velocities, distances, and time intervals involved. The availability of an accurate frequency standard within the vehicle will be of

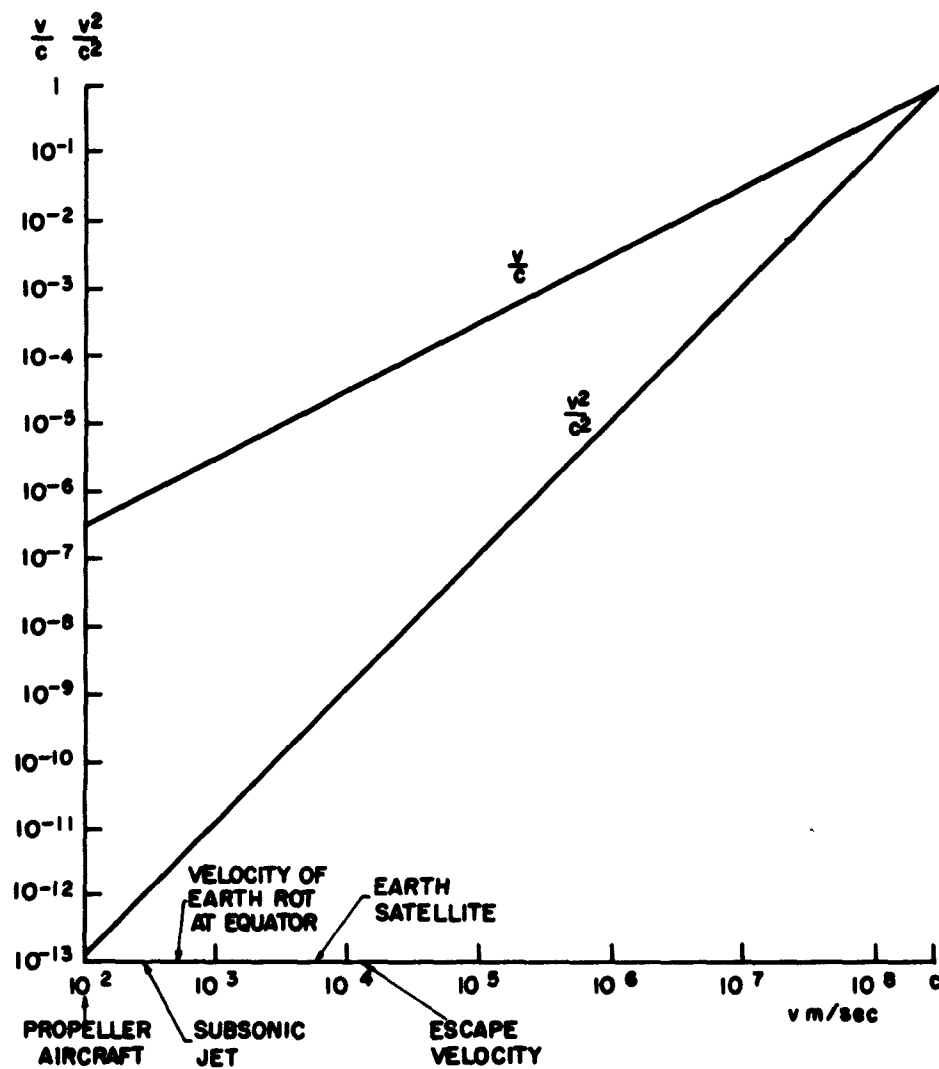


Figure 7

high value. Small frequency standards designed for airborne or space use unfortunately exhibit relatively low long-term frequency stability. If, for instance, a long-term frequency stability of  $1 \times 10^{-11}$  is required, one would have to employ a molecular or atomic frequency standard in a carefully controlled environment, protected from shock, vibration, and variations in temperature, barometric pressure, and magnetic and gravitational fields. Such frequency standards are definitely not suitable, at least at present, for use in space vehicles. One would therefore derive such an accurate and stable frequency from a ground-based standard and attempt to transmit it to the vehicle.

First-order Doppler shift can be cancelled through the use of the earlier described circuit. Station A must now be used in the vehicle, and station B on the ground. The ground standard frequency  $f$  is derived from an atomic frequency standard. At the vehicle, the frequency  $f$  is generated by an oscillator which does not have to be particularly stable. We shall consider again the simple circuit of Figure 1. The frequency generated at, and transmitted from, station A is varied until it is equal to  $f_3$ , the frequency received at A. It is thus necessarily equal to the standard frequency  $f$  generated at station B by the atomic frequency standard.

This oscillator-tuning operation at station A can be performed automatically if an error signal is derived from the difference between the transmitted frequency and  $f_3$ . It is then used to tune the vehicle-borne oscillator until the error signal vanishes. A circuit designed to perform this operation is depicted in Figure 8. The circuit is capable of transmitting a frequency to a moving vehicle while eliminating first-order Doppler. Second-order Doppler, in the illustrated example of  $v = 50,000$  m/sec (see Figure 8), is as large as  $3 \times 10^{-8}$ , or 3,000 times greater than the accuracy with which the frequency should be transmitted to the vehicle.

To reduce or eliminate the  $\frac{v^2}{c^2}$  term to reach the required accuracy of  $1 \times 10^{-11}$ , it is necessary that the vehicle velocity  $v$  (the velocity relative to the earth) is known; in this case, to an accuracy of 1 part in 3,000. If  $v$  is known, then a bias signal proportional to  $\frac{v^2}{c^2}$  can be generated and inserted into the AFC loop in Figure 8, as indicated in Figure 9.

The circuit of Figure 9 may seem rather complex, especially if it includes the necessary modifications as outlined on the earlier pages of this paper (refer to Figure 4), plus amplifiers, filters, power supplies, etc. This apparent complexity is outweighed by the important fact that a simple rugged oscillator with frequency control can be substituted for a bulky atomic frequency standard. Considering the fact that a frequency standard with a long-term stability of one part in  $10^{11}$  is not even suitable for vehicle use, this circuit makes the application

of stable frequency sources to vehicle electronic systems possible to degrees of stability which could not be achieved previously.

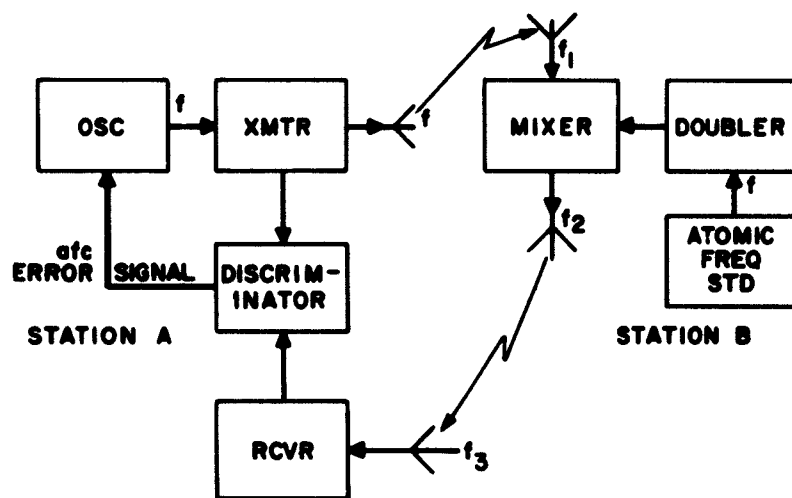


Figure 8

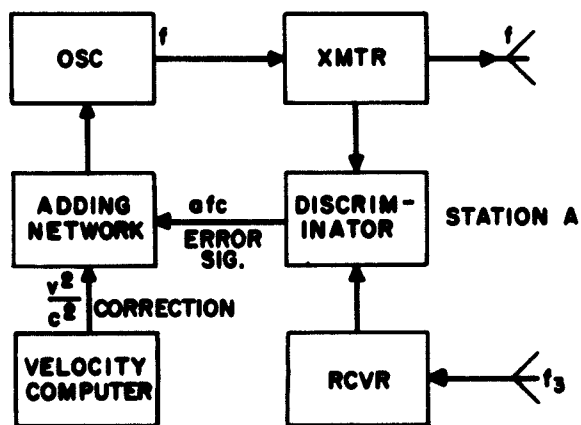


Figure 9

The figure of 1 part in  $10^{11}$  used in the above example can certainly be exceeded. If, for instance, an accuracy of 1 part in  $10^{12}$  is desired, the accuracy to which the relative vehicle velocity would have to be known, and the  $\frac{v^2}{c^2}$  bias signal be generated, would be one part in 30,000.

The author is aware of the fact that second-order Doppler shift has never been measured or even been proved to exist. It is assumed at this point that by the time it becomes significant in a space vehicle electronic system, its magnitude as function of velocity will be known, and the computer can be programmed accordingly.

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## References

1. C. Searle, Technical Memorandum No. 2, Research Laboratory of Electronics, Massachusetts Institute of Technology, June 15, 1959 (revised September 23, 1959).
2. Richard M. Waetjen, Cancellation of Doppler Frequency Shift, AFCRL Invention No. 8937, October 1961 (Patent Applied For).
3. ———. Doppler Navigation and Tracking (to be published).
4. ———. The AFCRL Gravitational Redshift Experiment (to be published).
5. R. V. Pound and G. A. Rebka, Jr., Apparent Weight of Photons, *Physical Review Letters* 4, p. 337, April 1, 1960.

<p>AF Cambridge Research Laboratories, Bedford, Mass.</p> <p><b>CANCELLATION OF DOPPLER FREQUENCY SHIFT</b>, by Richard M. Waetjen. January 1963. 23 p. incl. illus. AFCRL-63-70</p> <p>Unclassified report</p> <p>A Doppler cancellation scheme is presented which has potentialities in the fields of communication, data transmission, and navigation systems for vehicles moving at high velocities. The system is capable of continuous-wave operation. Received and transmitted frequencies at each of the stations involved are different, thus preventing feedback from transmitting to receiving antenna at the station.</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> <li>1. Navigation and Guidance</li> <li>2. Doppler Navigation and Doppler Systems</li> <li>3. Space Navigation</li> </ol> <p>1. Waetjen, R. M.</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> <li>1. Navigation and Guidance</li> <li>2. Doppler Navigation and Doppler Systems</li> <li>3. Space Navigation</li> </ol> <p>1. Waetjen, R. M.</p>
<p>AF Cambridge Research Laboratories, Bedford, Mass.</p> <p><b>CANCELLATION OF DOPPLER FREQUENCY SHIFT</b>, by Richard M. Waetjen. January 1963. 23 p. incl. illus. AFCRL-63-70</p> <p>Unclassified report</p> <p>A Doppler cancellation scheme is presented which has potentialities in the fields of communication, data transmission, and navigation systems for vehicles moving at high velocities. The system is capable of continuous-wave operation. Received and transmitted frequencies at each of the stations involved are different, thus preventing feedback from transmitting to receiving antenna at the station.</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> <li>1. Navigation and Guidance</li> <li>2. Doppler Navigation and Doppler Systems</li> <li>3. Space Navigation</li> </ol> <p>1. Waetjen, R. M.</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> <li>1. Navigation and Guidance</li> <li>2. Doppler Navigation and Doppler Systems</li> <li>3. Space Navigation</li> </ol> <p>1. Waetjen, R. M.</p>

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